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EXECUTIVE SUMMARY

A CRITICAL REVIEW ON THE APPLICATION OF ELASTIC-PLASTIC FRACTURE
MECHANICS TO NUCLEAR PRESSURE VESSEL AND PIPING SYSTEMS

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Conventional Linear-Elastic Fracture Mechanics (LEFM) techniques are based on the assumption that the material behaviour is brittle. Hence, Elastic-Plastic Fracture Mechanics (EPFM) techniques have been developed to account for situations in which extensive ductile behaviour may occur prior to failure. These explicitly treat the regime where extensive plastic deformation and stable crack growth may occur prior to fracture instability. This report provides a critical review of current research on the development and application of these new fracture mechanics techniques.

The methodology of EPFM is based on a comparison between the elastic-plastic crack driving force and a material-dependent resistance curve. Approaches using the J-integral as the crack driving force have clearly predominated. Currently, for a limited number of 2-D geometries, "Handbook" estimation formulae are available for determining the elastic-plastic J-integral, as an alternative to time consuming finite-element analyses.

Graphical EPFM evaluation methods, such as the J-integral/ Tearing Modulus (J/T) approach and the Failure Assessment Diagram (FAD), can provide a convenient means for comparing the crack driving force estimates with a resistance curve determined from standard test specimens. The FAD approach is more convenient and simpler to use in determining the safety factor based on load of a flawed structure. However, the J/T approach provides more detailed information on the stability of a flawed structure with regard to increasing load and crack growth.



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Report No 85-257-K

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ABSTRACT

A comprehensive literature survey on the application of Elastic-Plastic Fracture Mechanics to the assessment of the structural integrity of nuclear pressure vessels and piping is presented. In particular, the J-integral/Tearing Modulus (J/T) approach and the Failure Assessment Diagram (FAD) are covered in detail because of their general suitability for use in Ontario Hydro.

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NOMENCLATURE

a	Crack length
a _e	Effective crack length including plastic zone correction
Δa	Finite increment in crack length
c	Length of uncracked ligament
C _M	System compliance
COA	Crack opening angle
CTOA	Crack-tip opening angle
E	Young's modulus
E'	= E for plane stress, $= \frac{E}{(1-\nu^2)}$ for plane strain
J	Elastic-plastic crack driving force J-integral
J _e	Elastic portion of J
J _p	Plastic portion of J
$\hat{J}, \bar{J}$	Expressions for J with load factored out
J _R	Material resistance J-integral
K _I	Mode I linear-elastic stress intensity factor
K _R	Elastic stress intensity factor normalized by the material resistance stress intensity factor
n	Work-hardening exponent
P	Applied load
P ₀	Limit load
r	Local radial coordinate at the crack tip

S_r	Applied load normalized by the limit load
S_{ij}	Deviatoric stress components
SF_p	Safety factor based on load
T_J	Crack driving force tearing modulus
T_{JR}	Material resistance tearing modulus
δ	Denotes a virtual increment, or the crack opening displacement
Δ	Load line displacement (due to crack)
Δ_T	Total displacement of the system
ϵ_0	Yield strain
$\bar{\epsilon}_p$	Effective plastic strain
ϵ_{ij}^p	Plastic strain components
θ	Local angular coordinate at the crack tip
ν	Poisson's ratio
σ_0	Yield stress
σ^∞	Remote applied stress
$\bar{\sigma}_e$	Effective stress
σ_{ij}	Stress components

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To Mr. B. Montford
Manager, CNS
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A CRITICAL SURVEY ON THE APPLICATION OF ELASTIC-PLASTIC FRACTURE MECHANICS TO NUCLEAR PRESSURE VESSEL AND PIPING SYSTEMS

1.0 INTRODUCTION

The Fracture Mechanics Unit of the Research Division has conducted a number of structural integrity evaluations of nuclear components using the Linear-Elastic Fracture Mechanics (LEFM) method recommended in the ASME Code (ASME Boiler and Pressure Vessel Code, Section XI, Appendix A) for submission to the regulatory authorities (AECB and MCCR). The use of LEFM is based on the assumption that this approach, combined with a specified safety factor, is conservative.

However, in much or all of the working temperature regimes of nuclear systems the material is being stressed above the brittle-to-ductile transition temperature, where the fracture response is ductile and the material may be capable of considerable plastic deformation. The application of Elastic-Plastic Fracture Mechanics (EPFM) to such cases may result in more accurate assessments and hence less stringent safety factor requirements.

Recent analytical, experimental, and computational studies on ductile fracture have in fact demonstrated that a more realistic measure of fracture behaviour and safety margin may be obtained through the use of elastic-plastic analyses. With large international interest in EPFM, especially the US-NRC's adoption of the EPFM-based 'leak-before-break' design concept^{1,2/} for nuclear piping systems, Hydro may well be asked to conduct an EPFM analysis for a specific case by the regulatory authorities.

The objective of this report is to provide a review and critique of current research leading toward the development and application of EPFM techniques for nuclear pressure vessel and piping systems.

Some basic concepts on EPFM will first be presented. The selection of suitable criteria for EPFM will then be reviewed. This will be followed by a description of the generally accepted graphical EPFM evaluation methods. Some topics requiring further research will also be discussed.

2.0 BASIC CONCEPTS

2.1 Comparison Between Brittle and Ductile Crack Analysis

Brittle fracture is typified by rapid unstable crack propagation following a state of crack initiation. However, the progress of ductile fracture from an existing sharp-tipped flaw may be separated into four stages: the blunting of the initially sharp crack tip, crack growth initiation, stable crack growth, and unstable crack propagation. The plastic deformation associated with ductile fracture may in fact extend across the entire ligament of the cracked structure. The analytical techniques used for ductile fracture (EPFM) often make use of concepts developed originally for elastic fracture mechanics.

For example, a frequent assumption is that the material behaviour can be characterized by the deformation theory of plasticity/2,6-8/. This theory is based on the assumption that the stress components retain the same proportionality throughout the loading, such that a given stress component σ_{ij} can be written as

$$\sigma_{ij} = B \sigma_{ij_0} , \quad (2.1)$$

where B increases with load and σ_{ij_0} is some reference state of stress, such as the initial stress state. The use of equation (2.1) can result in a trivial integration of the incremental plasticity flow rules. One model is given as/24/,

$$\epsilon_{ij}^p = \frac{3}{2} \frac{S_{ij}}{\bar{\sigma}_e} \bar{\epsilon}_p , \quad (2.2)$$

where S_{ij} are the deviatoric stress components given by

$$\begin{aligned} S_{ij} &= \sigma_{ij} - \frac{1}{3} (\sigma_{11} + \sigma_{22} + \sigma_{33}) \text{ for } i = j, \\ S_{ij} &= \sigma_{ij} \text{ for } i \neq j. \end{aligned} \quad (2.3)$$

Here ϵ_{ij}^p are the plastic strain components, and $\bar{\epsilon}_p$ and $\bar{\sigma}_e$ are the effective plastic strain and effective stress, respectively. The use of equation (2.2) allows one to obtain solutions to fully plastic problems by using the same techniques as for non-linear elastic behaviour. By assuming deformation plasticity the near-tip stress and strain field can be made proportional to a single parameter, analogous to the stress intensity factor, K , in LEFM. The assumption of deformation plasticity is strictly valid only in the case of proportional loading for which the magnitude of the stress components increase in a proportional fashion, with no unloading.

2.2 J-Integral

One candidate to characterize the near-tip field under the assumption of deformation plasticity is the J-integral^{3/}, given by the following integration along the counter-clockwise contour Γ which joins with the lower and upper crack faces as shown in Figure 1.

$$J = \int_{\Gamma} (W \cdot n_1 - T_i \cdot \frac{\partial u_i}{\partial x_1}) ds. \quad (2.4)$$

In this equation W is the strain energy density, n_1 is the direction cosine of the normal to Γ with the x_1 -axis, T_i is the stress vector on Γ and u_i are the displacements. J is the rate of flow of energy to the crack-tip region per unit crack extension, and thus is a measure of the energy dissipation in the fracture process region surrounding the crack tip. Under the assumption of deformation plasticity J is integration-path independent. This not only means that J is unique for a given crack situation, but also allows some ease in computing a numerical value for J .

By using deformation plasticity the near-tip stresses can be written in terms of J as^{2/}

$$\sigma_{ij} = \left(\frac{J}{r}\right)^{1/(n+1)} F_{ij}(\theta, n) \quad (2.5)$$

where (r, θ) are polar coordinates originating from the crack tip (see Figure 1) and n is a hardening exponent. Equation (2.5) illustrates that the assumption of deformation plasticity allows one to characterize the intensity of the stress/strain field in

the same manner as K does in LEFM. Due to the nature of the near-tip singular field and the assumption of deformation plasticity, J will dominate the field in a limited region surrounding the crack tip under certain geometrical constraints. As well, for the J -integral to control the crack growth processes the amount of gross plasticity in the structure is also limited. These important limitations for J dominance and J -controlled growth are found in/2,8/ and presented in Section 4.2. Since deformation plasticity excludes elastic unloading, only small amounts of crack growth can usually be tolerated in using J (see/2/).

2.3 Structural Loading Characteristics

Real structures containing a crack (experimental testing machines, pressure vessels, etc) can be idealized as being comprised of a cracked region linked in series with a spring, representing the remainder of the structure, denoted as the system. This is illustrated in Figure 2. The cracked region contains a crack of length a , undergoes a displacement Δ due to applied load P . The system is subjected to the same load P and undergoes a displacement Δ_T (relative to the bottom fixed point) which is a function of the compliance of the system C_M , and the compliance of the cracked region. If C_M is near 0 (such as a large tensile testing machine) the loading condition on the cracked region is displacement controlled. If C_M is very large (such as a dead load or a pressurized pipe) the loading condition is load controlled. Many structures will lie between these two extremes.

The concepts of displacement-controlled and load-controlled conditions will be shown to be of great importance in the analysis of crack-growth stability.

3.0 SELECTION OF EPFM CRITERIA

3.1 Requirements

In LEFM crack initiation predictions are made by comparing the linear-elastic stress intensity factor, K , with the critical value for the material, K_c . Following the same approach, initial efforts in EPFM research were aimed at selecting a fracture criterion for characterizing crack initiation. In addition, further effort was aimed at obtaining stable crack growth and crack-growth instability criteria. The requirements used in selecting the criteria included the following/6/:

1. must be a measure of the stress and deformation field near the crack tip;

2. valid for fully plastic material behaviour;
3. critical value(s) independent of geometry and initial crack length;
4. valid for both small and large scale crack extensions;
5. able to predict crack initiation;
6. able to predict crack-growth instability;
7. well suited for models of three-dimensional crack fronts;
8. insensitive to numerical method of solution;
9. economical to use;
10. can be measured from small laboratory specimens in a tractable manner.

3.2 Methodology

The General Electric Company/6/ and the Battelle Columbus Laboratories/7/ conducted extensive research under the funding of the EPRI. The selection process consisted of three stages. In the first stage tests were conducted to obtain data on crack initiation and stable crack growth. In the second stage "Generation-Phase" analyses were performed in which experimentally-observed applied stress/stable crack growth behaviour was reproduced by using a finite-element model. Each of a number of candidate crack initiation and stable crack growth criteria were calculated from the field solution. The result was a collection of various crack growth resistance parameters for that particular material. The third stage was the "Application Phase" in which simulations were run of experiments by using a finite-element model and in turn, each of the candidate criteria, to predict the applied load/crack growth behaviour. The relationship between the Generation Phase and the Application Phase is illustrated in Figure 3.

3.3 Discussion of Criteria

The fracture criteria examined initially are given in Table 1 (see ref/2/ for a description of all these parameters), but attention was focussed on the J-integral and the crack opening angle.

The advantages of the J-integral are; its insensitivity to finite element type or mesh size; the computational ease of

evaluating J (in 2-D problems); and, because of its history independence for deformation plasticity, solutions for various situations can be catalogued for general use. However, while the J -integral is widely accepted as a criterion for crack initiation, the limitations imposed in using J to ensure proportional loading (see Sections 2.1, 2.2) usually restrict its application to problems involving only small amounts of stable crack growth.

The crack opening angle is appealing because of its readily-grasped physical meaning. There are two definitions of crack opening angle; a Crack-Tip Opening Angle (CTOA) that is a measure of the actual slopes of the crack faces at the crack tip; and an average Crack Opening Angle (COA) based on the crack opening displacement at the original crack tip location and the length of crack growth. These definitions are illustrated in Figure 4. Although the COA can be easily measured, its physical connection with the fracture process at the tip of the crack is not clear. Conversely, while the critical value of the CTOA can be associated with the fracture process at the crack tip, it presents a formidable measuring task. In addition, there are clearly some difficulties in applying either to mixed character shear/flat crack growth.

A number of investigators generalized the LEFM energy release rate (G) as a basic EPFM criterion. Their reasoning was that a proper stable crack growth criterion must delineate the energy dissipated in the direct fracture-related processes near the crack tip. All the generalized energy release rate parameters are basically the same, and are based on the work required to separate the crack faces per unit crack extension. However, these parameters are computational step-size dependent. Thus the numerical discretization error is a major influence on the computed critical values of the generalized energy release rates. This makes these parameters unattractive to use, in view of the fact that the J -integral, which is insensitive to computation step size, is also a measure of the energy dissipated in the fracture process region.

The consensus of the research/6,7/ is that a number of different criteria can be used as the basis of EPFM methodology to predict crack growth and instability using a resistance curve approach. But of all the EPFM criteria examined, J and CTOA appear to have the most potential. At the same time, both J and CTOA have significant advantages and disadvantages. As evidenced by experimental and analytical work on laboratory test specimens, J is computationally convenient and has a more definite theoretical basis, but it exhibits geometry dependence after a small amount of crack growth. On the other hand, CTOA exhibits a constant resistance value after an initial transient which makes it

highly attractive when large amounts of crack growth (stable or unstable) must be considered. But, its experimental determination is difficult and a theoretical basis is currently lacking.

4.0 THE J-INTEGRAL/TEARING MODULUS (J/T) APPROACH

In general, the calculations necessary to apply the EPFM methodology described in Section 3 require very sophisticated analyses, usually accomplished with the aid of advanced finite-element methods. To overcome difficulties in performing such an analysis, an engineering approach based on a J-estimation method was developed by the General Electric Company/8-10/ and led to the popular acceptance of the J/T stability assessment diagram approach/11-15/.

4.1 J_R-Resistance Curve

The use of the J_R-resistance curve in EPFM is largely due to the EPRI and the NRC. The J_R-resistance curve is analogous to the K_R(Δa) resistance (R) curve in LEFM. For any crack extension Δa, one equilibrium condition for stable crack growth is that the crack driving value of the J-integral due to the applied load, J(a,P), is equal to the material-resistance value, J_R(Δa).

$$J(a,P) = J_R(\Delta a) \quad (4.1)$$

The crack driving force J(a,P) is a function of crack length a and load P only, and the material resistance J_R(Δa) is only a function of the amount of crack extension, Δa.

4.2 The J/T Stability Assessment Diagram

The equilibrium condition given by equation (4.1) is not sufficient to ensure stable crack growth. The complete assessment of crack growth stability includes a second criterion; that unstable crack growth will occur when, for a given virtual crack increment δa, the increase in the crack driving force J(a,P) is greater than the increase in the material's resistance, J_R(Δa). Thus for a cracked structure,

$$\left(\frac{\partial J(a,P)}{\partial a} \right)_{\Delta_T} > \frac{dJ_R(\Delta a)}{da} \quad \text{for crack instability.} \quad (4.2)$$

The subscript Δ_T denotes the partial derivative with the total displacement Δ_T held constant.

Referring to Figure 2, an expression for the slope of the J-applied curve, $(\partial J(a,P)/\partial a)_{\Delta_T}$ can be obtained by writing

$$\Delta_T = \Delta + C_M P . \quad (4.3)$$

A first-order Taylor series expansion for a virtual increment δJ is

$$\delta J = \left(\frac{\partial J}{\partial a}\right)_P \delta a + \left(\frac{\partial J}{\partial P}\right)_a \delta P . \quad (4.4)$$

But for $\Delta_T = \text{constant}$, equation (4.3) can be used to express δP in terms of δa . Again use a first-order Taylor series expansion to obtain

$$\delta P = -\left(\frac{\partial \Delta}{\partial a}\right)_P \left[\left(\frac{\partial \Delta}{\partial P}\right)_a + C_M\right]^{-1} \delta a . \quad (4.5)$$

In this equation $(\partial \Delta / \partial P)_a$ is the compliance of the cracked region and C_M is the compliance of the system. Substitute equation (4.5) into (4.4) to obtain

$$\left(\frac{\partial J}{\partial a}\right)_{\Delta_T} = \left(\frac{\partial J}{\partial a}\right)_P - \left(\frac{\partial J}{\partial P}\right)_a \left(\frac{\partial \Delta}{\partial a}\right)_P \left[\left(\frac{\partial \Delta}{\partial P}\right)_a + C_M\right]^{-1} . \quad (4.6)$$

For a load-controlled system, $C_M \rightarrow \infty$, and equation (4.6) reduces to $(\partial J / \partial a)_{\Delta_T} = (\partial J / \partial a)_P$. Equations (4.1) and (4.2) can now be interpreted graphically for a load-controlled system by referring to Figure 5. The intersection of the J-applied curve (due to, say load P_1) with the J-resistance curve is an illustration of equation (4.1). When the slope of the J-applied curve equals the slope of the J-resistance curve (at load P_2), at the point of tangency, then equation (4.2) predicts that crack-growth instability will occur.

The derivatives in Equation (4.2) are written in non-dimensional form to define the crack driving tearing modulus,

$$T_J = \frac{E}{\sigma_o^2} \left(\frac{\partial J}{\partial a}\right)_{\Delta_T} \quad (4.7)$$

and the material resistance tearing modulus

$$T_{JR} = \frac{E}{2} \frac{dJ_R}{da} \quad (4.8)$$

where E is Young's modulus and σ_0 is a limit stress. Restated in terms of tearing moduli, equation (4.2) is

$$T_J \geq T_{JR} \quad \text{for instability.} \quad (4.9)$$

Having obtained the applied J and T_J as described above, one can plot these in the desired format to construct the so-called "stability assessment diagram"/8,12/. For example, one can plot J vs T_J curves with crack depth and C_M as parameters and superpose on these the material resistance (J_R vs T_{JR}) curve. The regimes of stable and unstable behaviour can be identified through the stability criterion, equation (4.9), as shown in Figure 6 where the T_{JR} curve defines the stable and unstable regions.

An important distinction between the $J_R(\Delta a)$ -resistance curve (for equilibrium) and the stability assessment diagram is that while the applied J is related only to local conditions in the cracked region, the applied tearing modulus is also dependent on the global system characteristics through C_M . For example, for displacement controlled loading ($C_M \approx 0$) a small amount of crack extension may result in a large reduction in P and hence the crack growth would be stable. For a load controlled situation ($C_M \rightarrow \infty$) a small amount of crack extension will result in no reduction in P and the crack growth may be unstable.

As mentioned in Section 2.2 the application of the J-integral to problems involving large-scale plasticity is limited by the criteria for a "J-dominated" field around the tip of the stationary crack and "J-controlled" crack growth. For validity, the J-dominated field must continue to surround the smaller fracture process region around the crack tip in which the mechanisms of void growth, void coalescence and elastic unloading occur. The criteria for J-controlled crack growth may limit the range of validity of J to small amounts of stable crack growth. For a fully yielded ligament these criteria are as follows/8/:

$$a) \quad \Delta a < 0.06 c, \quad (4.10)$$

where c is the length of the uncracked ligament.

b) Define

$$\omega = \frac{c}{J_{Ic}} \frac{dJ_R}{da}, \quad (4.11)$$

where J_{Ic} is the material's J-value at crack initiation.

For a component subjected primarily to tension,

$$\omega > 80 \quad (4.12)$$

and for a component subjected primarily to bending,

$$\omega > 10. \quad (4.13)$$

c) Define

$$\rho = \frac{c}{J_R/\sigma_o} \quad (4.14)$$

For a component subjected primarily to tension,

$$\rho > 200 \quad (4.15)$$

while for a component subjected primarily to bending,

$$\rho > 25.. \quad (4.16)$$

4.3 Estimation Procedure

An important consequence of the concept of J-controlled crack growth is that the J-integral crack driving force can be computed from analyses based on the deformation theory of plasticity. This has been employed as an underlying concept in GE's development of the engineering approach/8-10/. The key elements of the engineering approach are:

- i) a catalogue of fully plastic solutions for J-integral values;
- ii) an estimation procedure which enables the calculation of the elastic-plastic crack driving force by combining fully plastic solutions and elastic solutions;
- iii) simple graphical methods for predicting crack initiation, stable crack growth and instability by combining the crack driving force estimates with a resistance curve determined from standard specimen tests.

The estimation procedure provides an interpolation over an entire range of elastic-plastic deformation from small-scale yielding to fully yielded conditions including contained plasticity. In this, the elastic and fully plastic solutions are superposed to arrive at the elastic-plastic estimates. Estimation formulae are of the following form:

$$J(a, P) = J_e(a_e) + J_p(a, P), \quad (4.17)$$

where $J_e(a_e)$ is the elastic J-integral, $J_p(a, P)$ is the fully plastic J-integral and a_e is the effective crack length adjusted to include Irwin's small-scale yielding correction. The load P can be factored out of the terms in equation (4.17) with the result

$$J(a, P) = \hat{J}(a_e) \left[\frac{P}{P_0} \right]^2 + \hat{J}(a, n) \left[\frac{P}{P_0} \right]^{n+1}, \quad (4.18)$$

where P_0 is the limit load.

The results thus obtained have shown good agreement with detailed finite element analysis/8/. It should be noted that the approach yields accurate predictions for parameters of interest which would otherwise require elaborate finite-element computations. However, at the present tabulated solutions are available in /8,10/ only for two-dimensional or limited three-dimensional situations. Thus, solutions for three-dimensional problems may still require finite-element analysis.

5.0 NET-SECTION COLLAPSE CRITERION APPROACH

The net-section collapse criterion has long been recognized as a limiting case for LEFM fracture, where it was applied to subsize panels. Recently this concept was applied to circumferentially cracked pipes by Batelle/16/. This criterion assumes that failure occurs when the remaining ligament associated with the flawed section reaches the plastic collapse condition. Thus this criterion is applicable only to very ductile materials. The collapse load is determined by a limit load analysis where the yield stress is replaced in the collapse load expression by the flow stress σ_f which is usually chosen to fall between the yield and ultimate stresses.

To determine the flow stress, experiments are performed in which a remote stress σ is applied to a panel of width w containing a crack of length $2a$. The flow stress across the ligament at crack initiation is calculated from

$$\sigma_f = \frac{\sigma w}{w - 2a} \quad (5.1)$$

for different values of a . If σ_f at crack initiation is independent of a , then the net section flow stress can be used as a fracture criterion. The same method can also be used to obtain σ_f at the point of crack-growth instability.

Although simple to apply, the details of the fracture process considered in this investigation, namely, crack initiation, stable crack growth and instability are not factored in the net-section stress approach. The rationale for omitting such considerations is that the material is so ductile and tough that a section may fail due to plastic collapse prior to occurrence of the tearing instability event.

6.0 THE FAILURE ASSESSMENT DIAGRAM APPROACH

6.1 The R-6 Diagram

The R-6 failure assessment approach was developed mainly in the UK, derived from the Dowling and Townley two-criteria approach/17/. Harrison, Loosemore and Milne/18,19/ of the CEGB reformulated the two-criteria approach into a Failure Assessment Diagram (FAD). The diagram is based on the condition for crack failure being expressed by

$$J(a, \sigma^\infty) = J_R(\Delta a) \quad (6.1)$$

where $J(a, \sigma^\infty)$ is the J-integral due to the remote stress σ^∞ , obtained by using the Dugdale-Bilby-Cottrell-Swinden (DBCS) yield strip model,

$$J(a, \sigma^\infty) = \frac{8}{\pi} \sigma_0 \epsilon_0 a \ln \left[\sec \left(\frac{\pi}{2} \frac{\sigma^\infty}{\sigma_0} \right) \right] . \quad (6.2)$$

Here σ_0 and ϵ_0 are the yield stress and strain respectively. Equation (6.2) assumes a crack of length $2a$ in an infinite medium. By using equation (6.2) one writes the failure condition (equation (6.1)) in reciprocal form in terms of J :

$$\frac{1}{J_R(\Delta a)} = \frac{1}{\frac{8}{\pi} \sigma_0 \epsilon_0 a \ln \left[\sec \left(\frac{\pi}{2} \frac{\sigma^\infty}{\sigma_0} \right) \right]} \quad (6.3)$$

Substitute into equation (6.3) the following well-known expressions from LEFM:

$$K_I(a, \sigma^\infty) = \sigma^\infty \sqrt{\pi a} \quad ; \quad (6.4)$$

$$J(a, \sigma^\infty) = \frac{K_I(a, \sigma^\infty)^2}{E'} \quad ; \quad (6.5)$$

where $K_I(a, \sigma^\infty)$ is the linear-elastic stress intensity factor. Define the following non-dimensional parameters;

$$S_r = \frac{\sigma^\infty}{\sigma_0} \quad (6.6)$$

where σ_0 is a stress corresponding to a limit load; and

$$K_r = \frac{K_I(a, \sigma^\infty)}{K_R(\Delta a)} \quad ; \quad (6.7)$$

to arrive at (assuming $\sigma_0 = E' \epsilon_0$)

$$K_r^2 = \frac{S_r^2}{\frac{8}{\pi} \ln \left[\sec \left(\frac{\pi}{2} S_r \right) \right]} \quad (6.8)$$

Equation (6.8) defines a curve in the (K_r, S_r) plane, as shown in Figure 7. The curve defines various combinations of stress intensity factor and applied load which comprise the failure condition given by equation (6.1). The curve intersects the abscissa at $\sigma^\infty = \sigma_0$ since in equation (6.2) $J(a, \sigma^\infty) \rightarrow \infty$ as $\sigma^\infty \rightarrow \sigma_0$. Thus the R-6 diagram predicts that failure will always occur when the remote stress reaches the limit stress.

To assess a particular structure, the applied loading and elastic stress intensity factor are calculated, and the corresponding coordinates (K_R', S_R') are determined from equations (6.6) and (6.7). This point is then plotted on the R-6 diagram, as shown in Figure 7. If the point lies inside the curve the structure is predicted to be safe, otherwise the diagram predicts that failure will occur. Since K_R is linearly proportional to the load S_R , one can determine a safety factor based on load, SF_P , by evaluating the ratio of distances O-A' and O-B' (see Figure 7).

$$SF_P = \frac{O-B'}{O-A'} \quad (6.9)$$

The R-6 diagram can be characterized as predicting brittle failure for cases where $S_R > 0$, and predicting plastic collapse for cases where $S_R > 1.0$.

While the R-6 approach is simple to use, it suffers the disadvantage that it does not account for the geometry of the structure or the strain-hardening characteristics of the material.

6.2 The Deformation Plasticity Failure Assessment Diagram (DPFAD)

The R-6 approach has been extended by Kumar, German and Shih/8/ and by Bloom and Malik/20/ to account for structural geometry and material strain hardening. These effects can be readily incorporated into the failure assessment diagram by using the EPRI engineering-approach estimation formulae/8/ for deformation plasticity.

To obtain the Deformation Plasticity Failure Assessment Diagram (DPFAD), substitute the estimation formula for $J(a, P)$ given by equation (4.18) into the equilibrium condition, equation (4.1). The resulting expression in reciprocal form is

$$\frac{J_e(a, P)}{J_R(\Delta a)} = \frac{J_e(a, P)}{\hat{J}(a_e) \left[\frac{P}{P_0}\right]^2 + \hat{J}(a, n) \left[\frac{P}{P_0}\right]^{n+1}} \quad (6.10)$$

where both sides have been multiplied by the elastic J-integral, $J_e(a, P)$, given by

$$J_e(a, P) = \bar{J}(a) \left[\frac{P}{P_0}\right]^2. \quad (6.11)$$

Upon substituting equation (6.11) into equation (6.10) one has

$$\frac{J_e(a, P)}{J_R(\Delta a)} = \frac{\left[\frac{P}{P_0}\right]^2}{H_e \left[\frac{P}{P_0}\right]^2 + H_n \left[\frac{P}{P_0}\right]^{n+1}} \quad (6.12)$$

where

$$H_e = \frac{\hat{J}(a_e)}{\bar{J}(a)} \quad (6.13)$$

and

$$H_n = \frac{\hat{J}(a, n)}{\bar{J}(a)} \quad (6.14)$$

Define the following non-dimensional parameters

$$S_r = \frac{P}{P_0} \quad (6.15)$$

$$K_r = \frac{K_I(a, P)}{K_R(\Delta a)} \quad (6.16)$$

and

$$J_r = \frac{J_e(a, P)}{J_R(\Delta a)} \quad (6.17)$$

By employing equations (6.5) and (6.15-6.17) one writes (6.12) as

$$J_r = K_r^2 = \frac{S_r^2}{H_e S_r^2 + H_n S_r^{n+1}} \quad (6.18)$$

Equation (6.18) defines a curve in the (K_r, S_r) plane, as shown in Figure 8. As in the R-6 diagram, this curve defines a failure condition stated by equation (4.1). However, since equation (6.18) does not predict that $J(a, P) \rightarrow \infty$ as $P \rightarrow P_0$, the DPFAD curve does not approach the abscissa as $P \rightarrow P_0$ in

Figure 8. Also, implicit in equation (6.18) is a functional dependence of the DPFAD curve on crack length, although the dependence is usually insignificant/8/. Expressions used to construct the DPFAD are obtained from the "Handbook" J-estimation formulae found in /8,10/.

To assess the integrity of a structure, the applied load and elastic J-integral (or stress intensity factor) are calculated and substituted into equations (6.15) and (6.17) (or (6.16)), respectively. As in the R-6 approach, failure is predicted to occur if the point (K_R' , S_R') lies outside of the curve. Since the elastic stress intensity factor is also proportional to the applied load on the DPFAD, one can determine a safety factor based on load by using equation (6.9). The DPFAD is also characterized as an interpolation between the brittle failure mode ($S_R \rightarrow 0$) and plastic collapse ($S_R > 1.0$).

The DPFAD can be used for the assessment of crack initiation or the further growth of an extended crack. If failure is termed to occur at crack initiation, due to say cleavage, then one has

$$J_R(\Delta a = 0) = J_{Ic} ,$$

and $J_e(a_o, P)$ (or $K_I(a_o, P)$) and P_o are evaluated for the original crack length a_o . However, if failure is termed as the further extension of an extended crack of length $a_o + \Delta a$, due to ductile tearing, then $J_R(\Delta a)$ is obtained from the material J_R -resistance curve, and J_e and P_o are updated for the extended crack of length $a_o + \Delta a$.

7.0 AREAS REQUIRING FURTHER RESEARCH

7.1 Three-Dimensional Problems

Unlike LEFM, closed-form solutions for crack driving forces in the elastic-plastic range are available only for a relatively small number of simple geometries. Handbook style elastic-plastic J solutions, obtained by deformation plasticity finite-element analyses, are available in /8,10/ for 2-D geometries and straight cylindrical geometries. If structures containing flaws can be idealized as one of those cases given in /8,10/, then crack driving forces can be easily obtained.

Unfortunately, actual flaws found in plant components are located near pipe elbows, T-junctures, headers, etc. Under these circumstances, realistic J values can only be obtained by

performing a 3-D finite element analysis on a "case-by-case" approach.

Numerical EPFM analysis procedures are rather complicated, and a considerable amount of error can easily be introduced if the analysis is not done properly. This problem has been raised by various numerical round robin programs conducted under the auspice of the ASTM/21/, the European Group of Fracture/22/ and the Japanese/23/.

Since 1983 the Fracture Mechanics Unit has been developing a capability for obtaining J values numerically. A two-dimensional J evaluation (incremental plasticity) was performed using the general purpose finite-element code ABAQUS, and good agreement was shown against ASTM round robin results. In performing 3-D evaluations however, some difficulties were faced due to an algorithm error in the code. Resolving this problem will be a major task in 1985.

7.2 Elastic-Plastic Dynamic Crack Analysis

Such an analysis is required when the rate of loading on a cracked structure is such that the material inertia plays a significant role, such as a stress wave impinging on a stationary crack. Dynamic analysis is also required for predicting the propagation and arrest of rapidly moving crack fronts, whose speed may be of the order of the speed of sound in the material. Significant elastic unloading will occur due to stress wave interaction and the large amounts of crack growth. This invalidates the use of deformation plasticity theory. It is not surprising that most of the work to date has focussed on elastodynamic crack problems.

Of the research work done on elastic-plastic dynamic crack analysis, that by Emery *et al*/25/ is noteworthy in that they modelled dynamic crack propagation in a ductile pipe undergoing a depressurization transient. They employed a finite-difference method combined with CTOA as a fracture criterion. Much more effort is required to obtain solution techniques for such practical problems.

7.3 Residual Stresses

The introduction of residual stresses (due to welds, etc) greatly increases the complexity of the problem. Therefore, LEFM is usually applied to obtain a first estimate/2/. The problem is treated by linearly superposing the crack-tip stress fields due to the remote (primary) loading and the residual (secondary) loading. For example, in using a failure assessment

diagram, the plotted assessment points are calculated by using/20/

$$K_R' = K_R^{P'} + K_R^{S'} \quad (7.1)$$

and

$$S_R' = S_R^{P'}, \quad (7.2)$$

where $K_R^{P'}$ and $K_R^{S'}$ are the normalized linear-elastic stress intensity factors due to the primary and secondary stresses, respectively. Note that S_R' is still equal to the primary (remote) loading $S_R^{P'}$.

The effect of equation (7.1) is to change the $S_R' = 0$ intercept of the line 0-B' in Figures 7 and 8 to some (positive or negative) non-zero value.

8.0 CONCLUSIONS

A large amount of work has been devoted to the development and application of EPFM procedures; much of it motivated by nuclear safety considerations. The majority of these efforts are based on the use of the J-integral resistance curve. The primary reasons are:

- i) the direct fundamental connection between the J-integral and the near-tip stress field for ductile materials,
- ii) the catalogue capability of simple mathematical estimates of the J-integral without requiring the use of large-scale numerical computations.

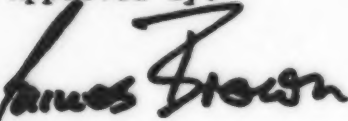
Graphical evaluation methods, such as the J/T approach and the Failure Assessment Diagram approach, can provide a convenient means for comparing the crack driving force estimates with a resistance curve determined from standard test specimens. The FAD approach is more convenient and simpler to use in a preliminary investigation of the safety margin of a flawed structure. In situations where the stability of a flawed structure is in question, the J/T approach provides a means for more detailed assessment of stability.

Further research efforts are required in the following areas:

- 1) three-dimensional J-integral computations;
- 2) elastic-plastic dynamic crack analysis;

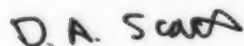
3) the effects of residual stresses in cracked components.

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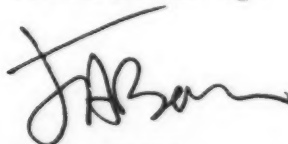


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TABLE 1
Candidate Fracture Criteria for the
Basis of a EPFM Methodology

Criterion Symbol	Identification	Designation of Critical Value
G	LEFM energy release rate	R
J	J-Integral	J_c
T	Tearing modulus	T_{mat}
G	Generalized energy release rate	R
COA	Average crack opening angle	$(COA)_c$
CTOA	Crack-tip opening angle	$(CTOA)_c$
F	Finite element crack tip Node force	F_c
G_o	Elastic-plastic energy	G_{oc}
G_z	Computational Process Zone energy Release rate	G_{zc}

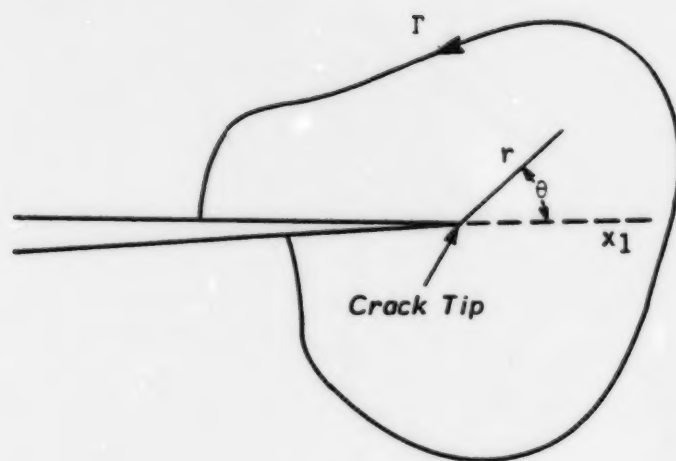


FIGURE 1
J-INTEGRAL CONTOUR SURROUNDING CRACK TIP

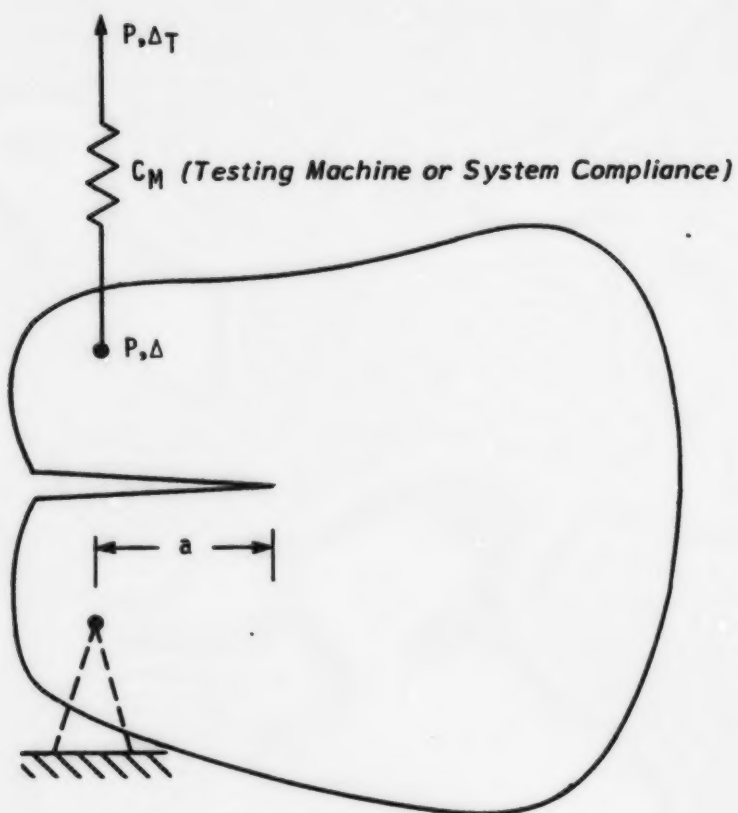


FIGURE 2
SCHEMATIC OF A CRACKED BODY.
THE SPRING REPRESENTS COMPLIANCE OF
THE SYSTEM (FROM/4/)

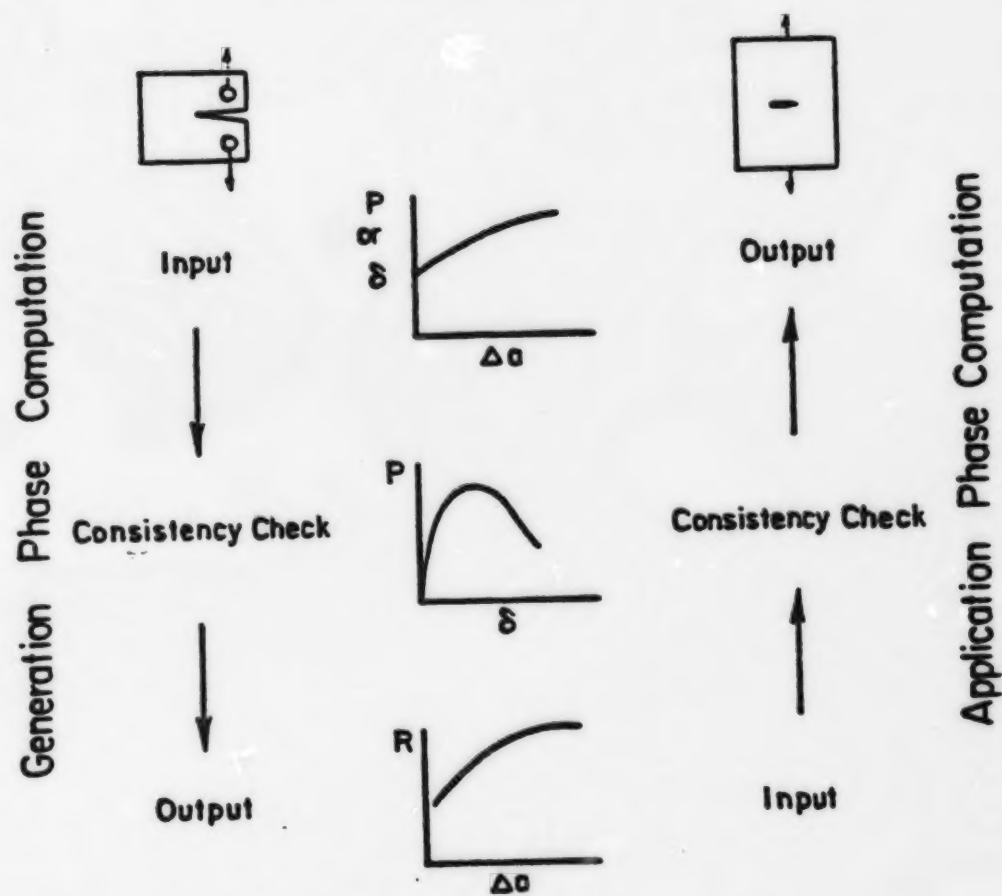


FIGURE 3
PROCEDURE FOLLOWED BY BATTELLE AND GE
FOR SELECTING EPFM CRITERIA

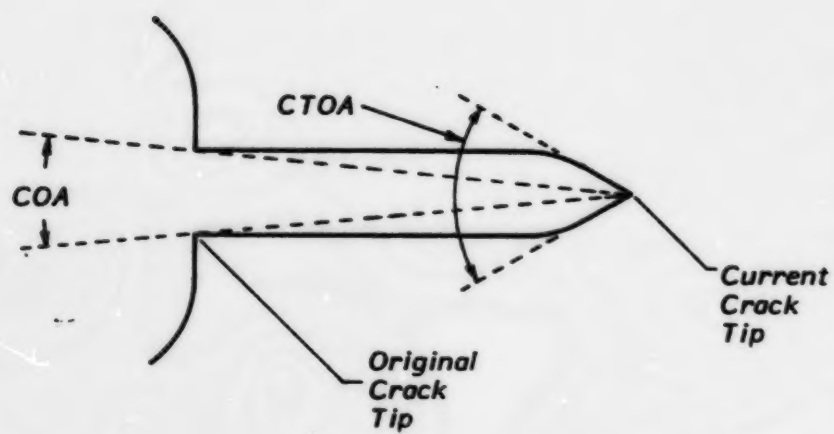


FIGURE 4
ILLUSTRATION OF DEFINITIONS OF COA AND CTOA

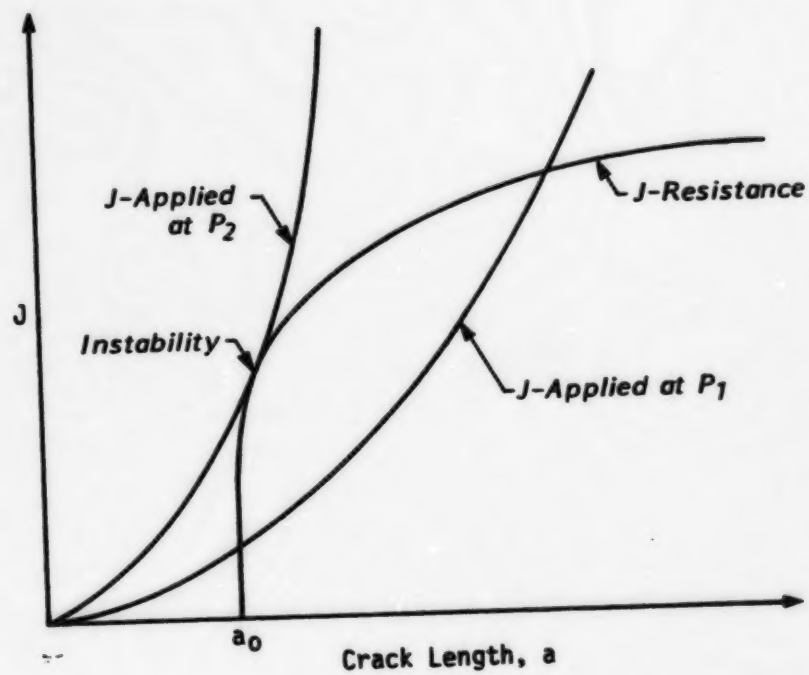


FIGURE 5
ILLUSTRATION OF J-INTEGRAL-BASED CRACK GROWTH
RESISTANCE/STABILITY CRITERIA

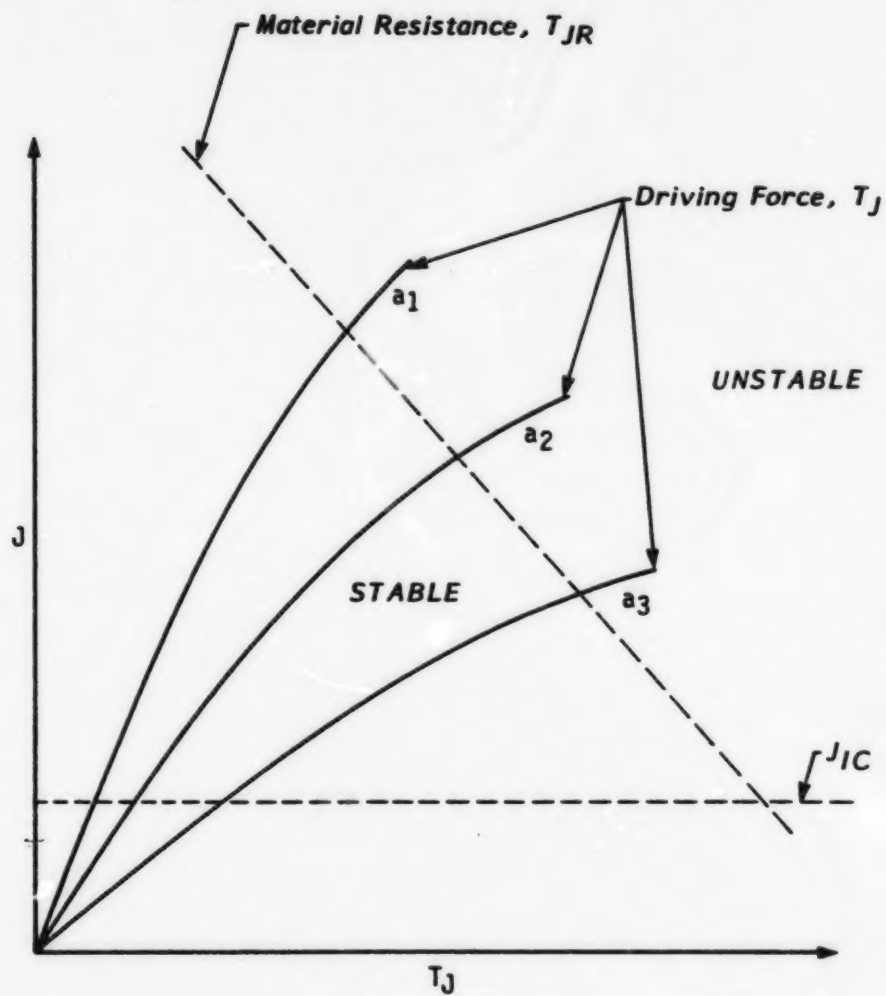


FIGURE 6
A SAMPLE J/T DIAGRAM

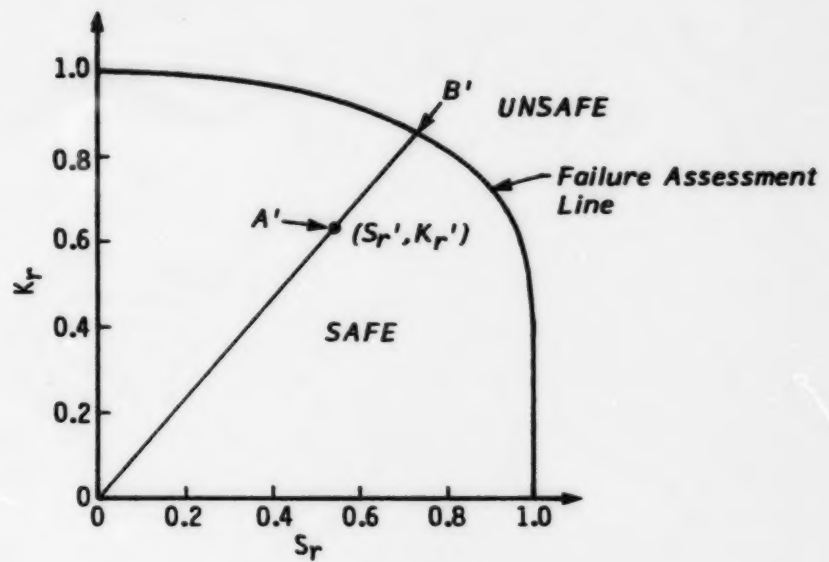


FIGURE 7
R-6 FAILURE ASSESSMENT DIAGRAM

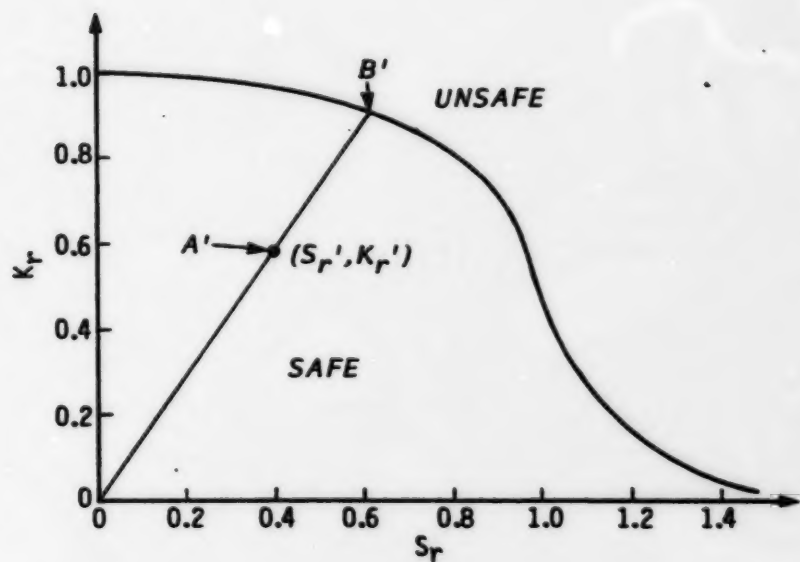


FIGURE 8
DEFORMATION PLASTICITY
FAILURE ASSESSMENT DIAGRAM